

## JEE-MAINS-2020-JAN-8-SHIFT-1

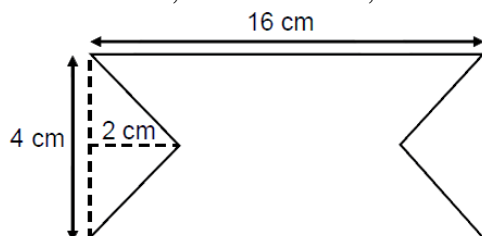
### PHYSICS

**Question 1:** Consider a uniform rod of mass  $M=4m$  and length  $l$  pivoted about its centre. A mass  $m$  moving with velocity  $v$  making angle  $\theta = \frac{\pi}{4}$  to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod - mass system just after the collision is:

**Options:**

- (a)  $\frac{4v}{7l}$
- (b)  $\frac{3\sqrt{2}v}{7l}$
- (c)  $\frac{3v}{7l}$
- (d)  $\frac{3v}{7\sqrt{2}l}$

**Question 2:** At time  $t = 0$  magnetic field of 1000 Gauss is passing perpendicularly through the area defined by the closed loop shown in the figure. If the magnetic field reduces linearly to 500 Gauss, in the next 5 s, then induced EMF in the loop is



**Options:**

- (a)  $28 \mu\text{V}$
- (b)  $36 \mu\text{V}$
- (c)  $48 \mu\text{V}$
- (d)  $56 \mu\text{V}$

**Question 3:** Proton with kinetic energy of 1 MeV moves from south to north. It gets an acceleration of  $10^{12} \text{ m/s}^2$  by an applied magnetic field (west to east). The value of magnetic field: (Rest mass of proton is  $1.6 \times 10^{-27} \text{ kg}$ )

**Options:**

- (a) 7.1 mT
- (b) 71 mT
- (c) 0.71 mT
- (d) 0.071 mT

**Question 4:** In finding the electric field using Gauss law the formula  $|\vec{E}| = \frac{Q_{enc}}{\epsilon_0 |A|}$  is

applicable. In the formula  $\epsilon_0$  is permittivity of free space, A is the area of Gaussian surface and  $q_{enc}$  is charge enclosed by the Gaussian surface. This equation can be used in which of the following situation?

**Options:**

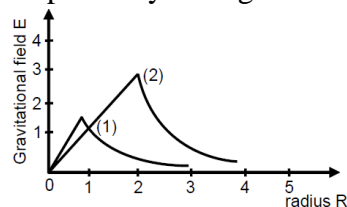
- (a) Only when the Gaussian surface is an equipotential surface.
- (b) Only when  $E$  = constant on the surface.
- (c) Only when the Gaussian surface is an equipotential surface and  $E$  is constant of the surface
- (d) For any choice of Gaussian surface

**Question 5:** Effective capacitance of parallel combination of two capacitors  $C_1$  and  $C_2$  is  $10 \mu\text{F}$ . When these capacitors are individually connected to a voltage source of  $1 \text{ V}$ , the energy stored in the capacitor  $C_2$  is 4 times that of  $C_1$ . If these capacitors are connected in series, their effective capacitance will be

**Options:**

- (a)  $4.2 \mu\text{F}$
- (b)  $8.4 \mu\text{F}$
- (c)  $3.2 \mu\text{F}$
- (d)  $1.6 \mu\text{F}$

**Question 6:** Consider two solid spheres of radii  $R_1 = 1 \text{ m}$   $R_2 = 2 \text{ m}$  and masses  $M_1$  and  $M_2$ , respectively. The gravitational field due to sphere 1 and 2 are shown. The value of  $M_1/M_2$  is



**Options:**

- (a)  $\frac{2}{3}$
- (b)  $\frac{1}{3}$
- (c)  $\frac{1}{2}$
- (d)  $\frac{1}{6}$

**Question 7:** A leak proof cylinder of length  $1 \text{ m}$ , made of a metal which has very low coefficient of expansion is floating vertically in water at  $0^\circ\text{C}$  such that its height above the water surface is  $20 \text{ cm}$ . When the temperature of water is increased to  $4^\circ\text{C}$ , the height of the

cylinder above the water surface becomes 21 cm. The density of water at  $T = 4^\circ\text{C}$ , relative to the density at  $T = 0^\circ\text{C}$  is close to

**Options:**

- (a) 1.03
- (b) 1.04
- (c) 1.26
- (d) 1.01

**Question 8:** The magnifying power of a telescope with tube length 60 cm is 5. What is the focal length of its eye piece?

**Options:**

- (a) 40 cm
- (b) 20 cm
- (c) 10 cm
- (d) 30 cm

**Question 9:** A particle of mass  $m$  is fixed to one end of a light spring having force constant  $k$  and unstretched length  $l$ . The other ends is fixed. The system is given an angular speed  $\omega$  about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is

**Options:**

- (a)  $\frac{ml\omega^2}{k + m\omega}$
- (b)  $\frac{ml\omega^2}{k + m\omega^2}$
- (c)  $\frac{ml\omega^2}{k - m\omega^2}$
- (d)  $\frac{ml\omega^2}{k - \omega m}$

**Question 10:** The graph which depicts and results of Rutherford gold foil experiment with  $\alpha$ -particles is

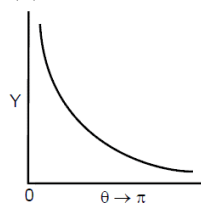
$\theta$  - Scattering angle

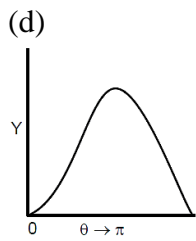
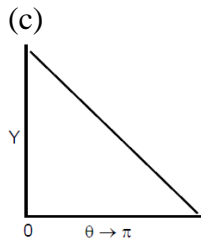
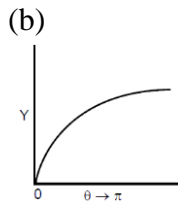
Y: Number of scattered  $\alpha$ -particles detected

(Plots are schematic and not to scale)

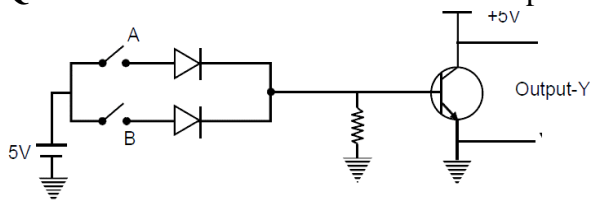
**Options:**

(a)





**Question 11:** Boolean relation at the output stage-Y for the following circuit is



**Options:**

- (a)  $\overline{A} + \overline{B}$
- (b)  $\overline{A} \cdot \overline{B}$
- (c)  $A \cdot B$
- (d)  $A + B$

**Question 12:** When photon of energy 4.0 eV strikes the surface of a metal A, the ejected photoelectrons have maximum kinetic energy  $T_A$  eV and de-Broglie wavelength  $\lambda_A$ . The maximum kinetic energy of photoelectrons liberated from another metal B by photon of energy 4.50 eV is  $T_B = (T_A - 1.5)$ eV. If the de-Broglie wavelength of these photoelectrons  $\lambda_B = 2\lambda_A$ , then the work function of metal B is

**Options:**

- (a) 2 eV
- (b) 3 eV
- (c) 1.5 eV
- (d) 4 eV

**Question 13:** The critical angle of a medium for a specific wavelength, if the medium has relative permittivity 3 and relative permeability  $4/3$  for this wavelength, will be

**Options:**

- (a)  $15^\circ$
- (b)  $30^\circ$
- (c)  $60^\circ$
- (d)  $45^\circ$

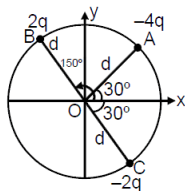
**Question 14:** Consider a solid sphere of radius  $R$  and mass density

$\rho(r) = \rho_0 \left(1 - \frac{r^2}{R^2}\right), 0 < r \leq R$ . The minimum density of a liquid in which it will float is

**Options:**

- (a)  $\frac{\rho_0}{3}$
- (b)  $\frac{2\rho_0}{3}$
- (c)  $\frac{\rho_0}{5}$
- (d)  $\frac{2\rho_0}{5}$

**Question 15:** Three charged particles A, B and C with charges  $-4q$ ,  $2q$  and  $-2q$  are present on the circumference of a circle of radius  $d$ . The charged particles A, C and centre O of the circle formed an equilateral triangle as shown in figure. Electric field at O along x-direction is:



**Options:**

- (a)  $\frac{\sqrt{3}q}{\pi\epsilon_0 d^2}$
- (b)  $\frac{\sqrt{3}q}{4\pi\epsilon_0 d^2}$
- (c)  $\frac{3\sqrt{3}q}{4\pi\epsilon_0 d^2}$
- (d)  $\frac{2\sqrt{3}q}{\pi\epsilon_0 d^2}$

**Question 16:** The dimension of stopping potential  $V_0$  in photoelectric effect in units of Planck's constant 'h', speed of light 'c' and Gravitational constant 'G' and ampere A is

**Options:**

- (a)  $h^{-2/3} c^{-1/3} G^{4/3} A^{-1}$

(b)  $h^{2/3} G^{5/3} c^{1/3} A^{-1}$

(c)  $h^{1/3} G^{2/3} c^{1/3} A^{-1}$

(d)  $h^{2/3} G^{3/2} c^{1/3} A^{-1}$

**Question 17:** The length of a potentiometer wire is 1200 cm and it carries a current of 60 mA. For a cell of emf 5 V and internal resistance of  $20 \Omega$ , the null point on it is found to be at 1000 cm. The resistance of whole wire is

**Options:**

(a)  $60 \Omega$

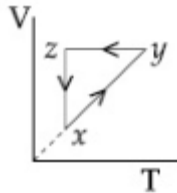
(b)  $120 \Omega$

(c)  $100 \Omega$

(d)  $80 \Omega$

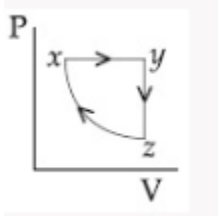
**Question 18:** The thermodynamic cycle  $xyzx$  is shown on a V-T diagram.

The P-V diagram that best describes this cycle is : (Diagrams are schematic and not to scale)

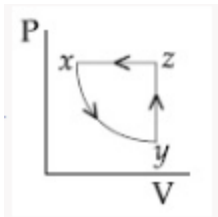


**Options:**

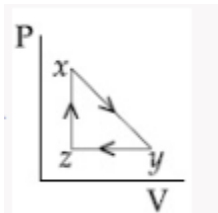
(a)



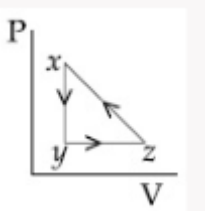
(b)



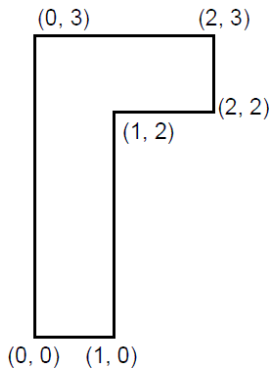
(c)



(d)



**Question 19:** The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4 kg. (The coordinates of the same are shown in figure) are



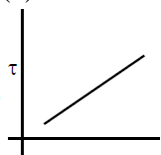
**Options:**

- (a) (1.25 m, 1.50 m)
- (b) (0.75 m, 0.75 m)
- (c) (0.75 m, 1.75 m)
- (d) (1 m, 1.75 m)

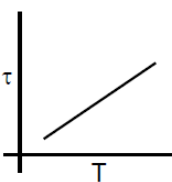
**Question 20:** The plot that depicts the behaviour of the mean free time ( $\tau$ ) (time between two successive collisions for the molecules of an ideal gas, as a function of temperature ( $T$ ), qualitatively, is : (Graphs are schematic and not drawn to scale)

**Options:**

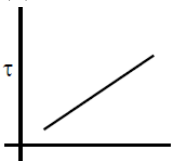
(a)



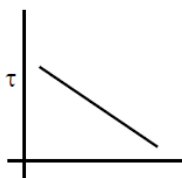
(b)



(c)



(d)



**Question 21:** A body A, mass  $m=0.1$  kg has an initial velocity of  $3\hat{i}$   $\text{ms}^{-1}$ . It collides elastically with another body, B of the same mass which has an initial velocity of  $5\hat{i}$   $\text{ms}^{-1}$ . The energy of B after collision is written as  $x/10$  J. The value of  $x$  is \_\_\_\_\_

**Question 22:** A particle is moving along the x-axis with its coordinates with time 't' given by  $x(t) = 10+8t-3t^2$ . Another particle is moving along the y-axis with its coordinate as a function of time given by  $y(t) = 5-8t^3$ . At  $t = 1$  s, the speed of the second particle measured in the frame of the first particle is given as  $\sqrt{v}$ . The  $v$  (in  $\text{m/s}$ ) is \_\_\_\_\_

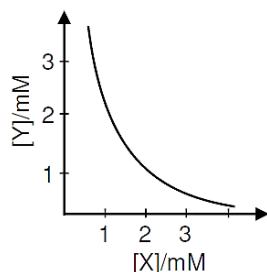
**Question 23:** A one metre long (both ends open) organ pipe is kept in a gas that has double the density of air at STP. Assuming the speed of sound in air at STP is 300  $\text{m/s}$ , the frequency difference between the fundamental and second harmonic of this pipe is \_\_\_\_\_ Hz.

**Question 24:** A point object is in front of the curved surface of a plano-convex lens. The radius of curvature of the curved surface is 30 cm and the refractive index of the lens material is 1.5, then the focal length of the lens (in cm) is \_\_\_\_\_

**Question 25:** Four resistances of  $15\Omega$ ,  $12\Omega$ ,  $4\Omega$  and  $10\Omega$  respectively in cyclic order to form Wheatstone's network. The resistance that is to be connected in parallel with the resistance of  $10\Omega$  to balance the network is \_\_\_\_\_  $\Omega$

## CHEMISTRY

**Question 26:** The stoichiometry and solubility product of a salt with the solubility curve given below is, respectively



### Options:

- (a)  $X_2Y$ ,  $2 \times 10^{-9} \text{ M}^3$
- (b)  $XY$ ,  $2 \times 10^{-6} \text{ M}^3$
- (c)  $XY_2$ ,  $4 \times 10^{-9} \text{ M}^3$



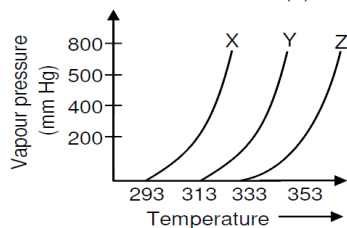
(d)  $XY_2$ ,  $1 \times 10^{-9} M^3$

**Question 27:** A graph of vapour pressure and temperature for three different liquids X, Y and Z is shown below

The following inferences are made

- (1) X has higher intermolecular interactions compared to Y.
- (2) X has lower intermolecular interactions compared to Y.
- (3) Z has lower intermolecular interactions compared to Y.

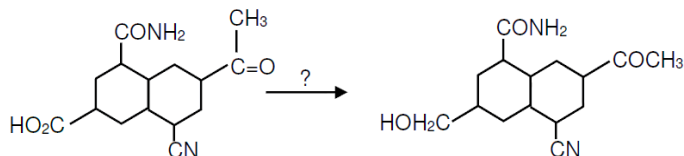
The correct inference(s) is/are



**Options:**

- (a) (C)
- (b) (B)
- (c) (A) and (C)
- (d) (A)

**Question 28:** The most suitable reagent for the given conversion is



**Options:**

- (a)  $H_2/Pd$
- (b)  $NaBH_4$
- (c)  $LiAlH_4$
- (d)  $B_2H_6$

**Question 29:** The complex that can show fac- and mer- isomers is

**Options:**

- (a)  $[CoCl_2(en)_2]$
- (b)  $[Pt(NH_3)_2Cl_2]$
- (c)  $[Co(NH_3)_4Cl_2]^+$
- (d)  $[Co(NH_3)_3(NO_2)_3]$

**Question 30:** The rate of a certain biochemical reaction at physiological temperature (T) occurs  $10^6$  times faster with enzyme than without. The change in the activation energy upon adding enzyme is

**Options:**

- (a)  $+ 6(2.303)RT$
- (b)  $+ 6RT$

- (c)  $-6RT$   
 (d)  $-6(2.303)RT$

**Question 31:** The first ionization energy (in kJ/mol) of Na, Mg, Al and Si respectively, are

**Options:**

- (a) 496, 577, 737, 786  
 (b) 786, 737, 577, 496  
 (c) 496, 577, 786, 737  
 (d) 496, 737, 577, 786

**Question 32:** The third ionization enthalpy is minimum for

**Options:**

- (a) Ni  
 (b) Co  
 (c) Mn  
 (d) Fe

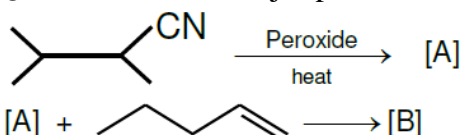
**Question 33:** Among the gases (a) – (e), the gases that cause greenhouse effect are

- (a)  $\text{CO}_2$                       (b)  $\text{H}_2\text{O}$                       (c) CFCs                      (d)  $\text{O}_2$     (e)  $\text{O}_3$

**Options:**

- (a) (a), (c), (d) and (e)  
 (b) (a), (b), (c) and (e)  
 (c) (a), (b), (c) and (d)  
 (d) (a) and (d)

**Question 34:** The major products A and B in the following reactions are



**Options:**

- (a)  
 A = and तथा B =
- (b)  
 A = and तथा B =
- (c)  
 A = and तथा B =
- (d)  
 A = and तथा B =

**Question 35:** The predominant intermolecular forces present in ethyl acetate, a liquid, are

**Options:**

- (a) Dipole-dipole and hydrogen bonding
- (b) London dispersion, dipole-dipole and hydrogen bonding
- (c) London dispersion and dipole-dipole
- (d) Hydrogen bonding and London dispersion

**Question 36:** Arrange the following compounds in increasing order of C–OH bond length  
Methanol, phenol, p-ethoxyphenol

**Options:**

- (a) methanol < phenol < p-ethoxyphenol
- (b) phenol < methanol < p-ethoxyphenol
- (c) phenol < p-ethoxyphenol < methanol
- (d) methanol < p-ethoxyphenol < phenol

**Question 37:** A flask contains a mixture of isohexane and 3-methylpentane. One of the liquids boils at 63°C while the other boils at 60°C. What is the best way to separate the two liquids and which one will be distilled out first?

**Options:**

- (a) simple distillation, isohexane
- (b) simple distillation, 3-methylpentane
- (c) fractional distillation, 3-methylpentane
- (d) fractional distillation, isohexane

**Question 38:** For the Balmer series in the spectrum of H atom,  $\bar{\nu} = R_H \left[ \frac{1}{n_1^2} - \frac{1}{n_2^2} \right]$  the correct

statements among (I) to (IV) are

- (I) As wavelength decreases, the lines in the series converge
- (II) The integer  $n_1$  is equal to 2
- (III) The lines of longest wavelength correspond to  $n_2 = 3$
- (IV) The ionization energy of hydrogen can be calculated from wave number of these lines

**Options:**

- (a) (I), (III), (IV)
- (b) (I), (II), (IV)
- (c) (II), (III), (IV)
- (d) (I), (II), (III)

**Question 39:** The number of bonds between sulphur and oxygen atoms in  $S_2O_8^{2-}$  and the number of bonds between sulphur and sulphur atoms in rhombic sulphur, respectively, are

**Options:**

- (a) 8 and 8
- (b) 8 and 6
- (c) 4 and 6
- (d) 4 and 8

**Question 40:** The strength of an aqueous NaOH solution is most accurately determined by titrating (Note consider that an appropriate indicator is used)

**Options:**

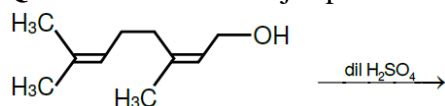
- (a) Aq. NaOH in a volumetric flask and concentrated H<sub>2</sub>SO<sub>4</sub> in a conical flask
- (b) Aq. NaOH in a burette and aqueous oxalic acid in a conical flask
- (c) Aq. NaOH in a pipette and aqueous oxalic acid in a burette
- (d) Aq. NaOH in a burette and concentrated H<sub>2</sub>SO<sub>4</sub> oxalic in a conical flask

**Question 41:** As per Hardy–Schulze formulation, the flocculation values of the following for ferric hydroxide sol are in the order

**Options:**

- (a) K<sub>3</sub>[Fe(CN)<sub>6</sub>] < K<sub>2</sub>CrO<sub>4</sub> < KBr = KNO<sub>3</sub> = AlCl<sub>3</sub>
- (b) K<sub>3</sub>[Fe(CN)<sub>6</sub>] < K<sub>2</sub>CrO<sub>4</sub> < AlCl<sub>3</sub> < KBr < KNO<sub>3</sub>
- (c) K<sub>3</sub>[Fe(CN)<sub>6</sub>] > AlCl<sub>3</sub> > K<sub>2</sub>CrO<sub>4</sub> > KBr > KNO<sub>3</sub>
- (d) AlCl<sub>3</sub> > K<sub>3</sub>[Fe(CN)<sub>6</sub>] > K<sub>2</sub>CrO<sub>4</sub> > KBr = KNO<sub>3</sub>

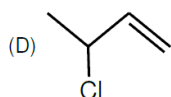
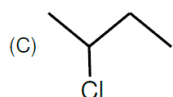
**Question 42:** The major product of the following reaction is



**Options:**

- (a)
- (b)
- (c)
- (d)

**Question 43:** The decreasing order of reactivity towards dehydrohalogenation (E1) reaction of the following compounds is



**Options:**

- (a)  $B > D > C > A$
- (b)  $D > B > C > A$
- (c)  $B > A > D > C$
- (d)  $B > D > A > C$

**Question 44:** When gypsum is heated to 393 K, it forms

**Options:**

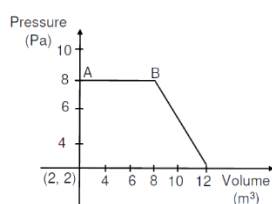
- (a) Anhydrous  $\text{CaSO}_4$
- (b)  $\text{CaSO}_4 \cdot 0.5 \text{H}_2\text{O}$
- (c)  $\text{CaSO}_4 \cdot 5 \text{H}_2\text{O}$
- (d) Dead burnt plaster

**Question 45:** Which of the following statement is not true for glucose

**Options:**

- (a) Glucose exists in two crystalline forms  $\alpha$  and  $\beta$
- (b) Glucose reacts with hydroxylamine to form oxime
- (c) Glucose gives Schiff's test for aldehyde
- (d) The pentaacetate of glucose does not react with hydroxylamine to give oxime

**Question 46:** The magnitude of work done by a gas that undergoes a reversible expansion along the path ABC shown in the figure is \_\_\_\_\_



**Question 47:** The volume (in mL) of 0.125 M  $\text{AgNO}_3$  required to quantitatively precipitate chloride ions in 0.3 g of  $[\text{Co}(\text{NH}_3)_6]\text{Cl}_3$  is \_\_\_\_\_.

$$M[\text{Co}(\text{NH}_3)_6]\text{Cl}_3 = 267.46 \text{ g/mol}$$

$$M\text{AgNO}_3 = 169.87 \text{ g/mol}$$

**Question 48:** Ferrous sulphate heptahydrate is used to fortify foods with iron. The amount (in grams) of the salt required to achieve 10 ppm of iron in 100 kg of wheat is \_\_\_\_\_.

$$\text{Atomic weight Fe} = 55.85 \quad \text{S} = 32.00 \quad \text{O} = 16.00$$

**Question 49:** What would be the electrode potential for the given half-cell reaction at pH = 5  
 $2\text{H}_2\text{O} \rightarrow \text{O}_2 + 4\text{H}^+ + 4\text{e}^-$   $E_0^{\text{red}} = 1.23 \text{ V}$  ( $R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1}$  Temp = 298 K oxygen under std. pressure of 1 bar)

**Question 50:** The number of chiral centres in penicillin is

### MATHEMATICS

**Question 51:**  $\lim_{x \rightarrow 0} \left( \frac{3x^2 + 2}{7x^2 + 2} \right)^{\frac{1}{x^2}}$  is equal to

**Options:**

- (a)  $1/e^2$
- (b)  $e$
- (c)  $e^2$
- (d)  $1/e$

**Question 52:** Let two points be A(1, -1) and B(0, 2). If a point P(x', y') be such that the area of  $\Delta\text{PAB} = 5$  square. units and it lies on the line,  $3x + y - 4\lambda = 0$ , then a value of  $\lambda$  is :

**Options:**

- (a) 3
- (b) -3
- (c) 1
- (d) 4

**Question 53:** The mean and the standard deviation (s, d) of 10 observations are 20 and 2 respectively. Each of these 10 observations is multiplied by p and then reduced by q, where  $p \neq 0$  and  $q \neq 0$ . If the new mean and new s.d. become half of their original values, then q is equal to :

**Options:**

- (a) -20
- (b) 10
- (c) -10
- (d) -5

**Question 54:** The shortest distance between the lines

$$\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-8}{1} \text{ and } \frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4} \text{ is}$$

**Options:**

- (a)  $2\sqrt{30}$

- (b) 3
- (c)  $\frac{7}{2}\sqrt{30}$
- (d)  $3\sqrt{30}$

**Question 55:** If a, b and c are the greatest values of  ${}^P19C$ ,  ${}^Q20C$  and  ${}^R21C$  respectively, then :

**Options:**

- (a)  $\frac{a}{10} = \frac{b}{11} = \frac{c}{42}$
- (b)  $\frac{a}{10} = \frac{b}{11} = \frac{c}{21}$
- (c)  $\frac{a}{11} = \frac{b}{22} = \frac{c}{42}$
- (d)  $\frac{a}{11} = \frac{b}{22} = \frac{c}{21}$

**Question 56:** Let the volume of a parallelepiped whose coterminous edges are given by  $\vec{u} = \hat{i} + \hat{j} + \lambda\hat{k}$ ,  $\vec{v} = \hat{i} + \hat{j} + 3\hat{k}$  and  $\vec{w} = 2\hat{i} + \hat{j} + \hat{k}$  be 1 cu. unit. If  $\theta$  be the angle between the edges  $\vec{u}$  and  $\vec{w}$  then  $\cos\theta$  can be

**Options:**

- (a)  $\frac{5}{3\sqrt{3}}$
- (b)  $\frac{7}{6\sqrt{6}}$
- (c)  $\frac{5}{7}$
- (d)  $\frac{7}{6\sqrt{3}}$

**Question 57:** Let  $f(x) = x\cos^{-1}(-\sin|x|)$ ,  $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ , then which of the below is true ?

**Options:**

- (a)  $f'(0) = -\frac{\pi}{2}$
- (b)  $f'$  is decreasing in  $\left(-\frac{\pi}{2}, 0\right)$  and increasing in  $\left(0, \frac{\pi}{2}\right)$
- (c)  $f$  is not differentiable at  $x = 0$
- (d)  $f'$  is increasing in  $\left(-\frac{\pi}{2}, 0\right)$  and decreasing in  $\left(0, \frac{\pi}{2}\right)$

**Question 58:** Let  $f(x) = (\sin(\tan^{-1}x) + \sin(\cot^{-1}x))^2 - 1$ ,  $|x| > 1$ . If  $\frac{dy}{dx} = \frac{1}{2} \frac{d}{dx}(\sin^{-1}(f(x)))$

and  $y(\sqrt{3}) = \frac{\pi}{6}$ , then  $y(-\sqrt{3})$  is equal to :

**Options:**

- (a)  $\pi/3$
- (b)  $2\pi/3$
- (c)  $-\pi/6$
- (d)  $5\pi/6$

**Question 59:** Let A and B be two independent events such that  $P(A) = \frac{1}{3}$  and

$P(B) = \frac{1}{6}$ . Then, which of the following is TRUE?

**Options:**

- (a)  $P(A/B) = \frac{1}{3}$
- (b)  $P(A/B) = \frac{2}{3}$
- (c)  $P(A/B') = \frac{1}{3}$
- (d)  $P(A/(A \cup B)) = \frac{1}{4}$

**Question 60:** The inverse function of  $f(x) = \frac{8^{2x} - 8^{-2x}}{8^{2x} + 8^{-2x}}$ ,  $x \in (-1, 1)$ , is

**Options:**

- (a)  $\frac{1}{4} \log_e \left( \frac{1+x}{1-x} \right)$
- (b)  $\frac{1}{4} \log_e \left( \frac{1-x}{1+x} \right)$
- (c)  $\frac{1}{4} (\log_e e) \left( \frac{1+x}{1-x} \right)$
- (d)  $\frac{1}{4} (\log_e e) \left( \frac{1-x}{1+x} \right)$

**Question 61:** If c is a point at which Rolle's theorem holds for the function

$f(x) = \log_e \left( \frac{x^2 + \alpha}{7x} \right)$  in the interval  $[3, 4]$ , where  $\alpha \in \mathbb{R}$ , then  $f'(c)$  is equal to :

**Options:**



(a)  $\frac{\sqrt{3}}{7}$

(b)  $-\frac{1}{24}$

(c)  $-\frac{1}{12}$

(d)  $\frac{1}{12}$

**Question 62:** Let  $y = y(x)$  be a solution of the differential equation,

$$\sqrt{1-x^2} \frac{dy}{dx} + \sqrt{1-y^2} = 0, |x| < 1.$$

If  $y\left(\frac{1}{2}\right) = \frac{\sqrt{3}}{2}$ , then  $y\left(\frac{-1}{\sqrt{2}}\right)$  is equal to

(a)  $-\frac{\sqrt{3}}{2}$

(b)  $\frac{1}{\sqrt{2}}$

(c)  $\frac{\sqrt{3}}{2}$

(d)  $-\frac{1}{\sqrt{2}}$

**Question 63:** If  $\int \frac{\cos x \, dx}{\sin^3 x (1 + \sin^6 x)^{2/3}} = f(x)(1 + \sin^6 x)^{1/\lambda} + c$  where  $c$  is a constant of

integration, then  $\lambda f\left(\frac{\pi}{3}\right)$  is equal to :

**Options:**

(a) 2

(b)  $-\frac{9}{8}$

(c)  $\frac{9}{8}$

(d) -2

**Question 64:** Let the line  $y = mx$  and the ellipse  $2x^2 + y^2 = 1$  intersect at a point P in the first quadrant. If the normal to this ellipse at P meets the co-ordinate axes at  $\left(-\frac{1}{3\sqrt{2}}, 0\right)$  and

$(0, \beta)$ , then  $\beta$  is equal to :

**Options:**

(a)  $\frac{2}{\sqrt{3}}$

(b)  $\frac{\sqrt{2}}{3}$

(c)  $\frac{2}{3}$

(d)  $\frac{2\sqrt{2}}{3}$

**Question 65:** The locus of a point which divides the line segment joining the point  $(0, -1)$  and a point on the parabola,  $x^2 = 4y$ , internally in the ratio  $1 : 2$ , is :

**Options:**

(a)  $9x^2 - 12y = 8$

(b)  $x^2 - 3y = 2$

(c)  $9x^2 - 3y = 2$

(d)  $4x^2 - 3y = 2$

**Question 66:** Which one of the following is a tautology?

**Options:**

(a)  $(P \wedge (P \rightarrow Q)) \rightarrow Q$

(b)  $(P \wedge (P \vee Q))$

(c)  $Q \rightarrow (P \wedge (P \rightarrow Q))$

(d)  $(P \vee (P \wedge Q))$

**Question 67:** For  $a > 0$ , let the curves  $C_1: y^2 = ax$  and  $C_2: x^2 = ay$  intersect at origin  $O$  and a point  $P$ . Let the line  $x = b$  ( $0 < b < a$ ) intersect the chord  $OP$  and the  $x$ -axis at points  $Q$  and  $R$ , respectively. If the line  $x = b$  bisects the area bounded by the curves,  $C_1$  and  $C_2$ , and the area of  $\Delta OQR = \frac{1}{2}$ , then 'a' satisfies the equation :

**Options:**

(a)  $x^6 - 12x^3 - 4 = 0$

(b)  $x^6 - 6x^3 + 4 = 0$

(c)  $x^6 - 12x^3 + 4 = 0$

(d)  $x^6 + 6x^3 - 4 = 0$

**Question 68:** If the equation,  $x^2 + bx + 45 = 0$  ( $b \in \mathbb{R}$ ) has conjugate complex roots and they satisfy  $|z+1| = 2\sqrt{10}$ , then :

**Options:**

(a)  $b^2 - b = 30$

(b)  $b^2 + b = 72$

(c)  $b^2 - b = 42$

(d)  $b^2 + b = 12$

**Question 69:** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be such that for all  $x \in \mathbb{R}$   $(2^{1+x} + 2^{1-x})$ ,  $f(x)$  and  $(3^x + 3^{-x})$  are in A.P., then the minimum value of  $f(x)$  is :

**Options:**

- (a) 0
- (b) 1
- (c) 4
- (d) 3

**Question 70:** For which of the following ordered pairs  $(\mu, \delta)$ , the system of linear equations

$$x + 2y + 3z = 1$$

$$3x + 4y + 5z = \mu$$

$$4x + 4y + 4z = \delta \text{ is inconsistent ?}$$

**Options:**

- (a) (4, 6)
- (b) (3, 4)
- (c) (1, 0)
- (d) (4, 3)

**Question 71:** The number of all  $3 \times 3$  matrices  $A$ , with entries from the set  $\{-1, 0, 1\}$  such that the sum of the diagonal elements of  $AA^T$  is 3, is \_\_\_\_\_

**Question 72:** The least positive value of 'a' for which the equation,

$$2x^2 + (a - 10)x + \frac{33}{2} = 2a \text{ has real roots is}$$

**Question 73:** An urn contains 5 red marbles, 4 black marbles and 3 white marbles. Then the number of ways in which 4 marbles can be drawn so that at the most three of them are red is \_\_\_\_\_.

**Question 74:** The sum  $\sum_{k=1}^{20} (1 + 2 + 3 + \dots + k)$  is

**Question 75:** Let the normal at a point  $P$  on the curve  $y^2 - 3x^2 + y + 10 = 0$  intersect the  $y$ -axis at  $(0, 3/2)$ . If  $m$  is the slope of the tangent at  $P$  to the curve, then  $|m|$  is equal to